Introduction to Rule 110

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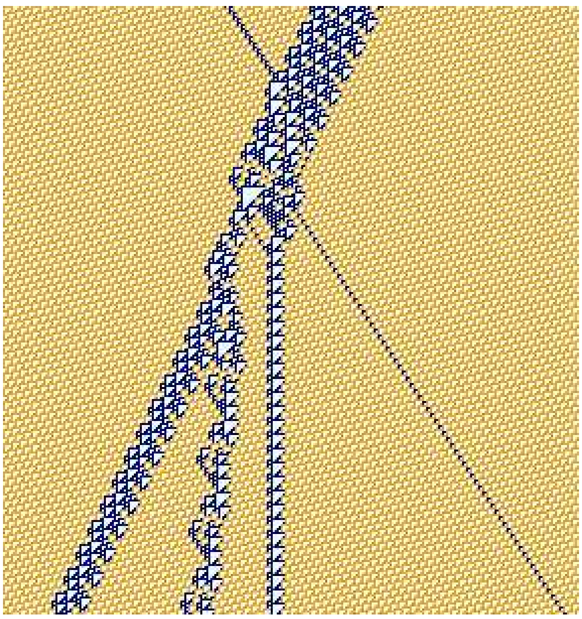
Abstract

A brief introduction to the study of the cellular automaton Rule 110 is presented. We begin with a general historical background of cellular automata theory, discussing the most important stages of development. Later we show the antecedents in the study of Rule 110, making special emphasis in the conjecture of Stephen Wolfram and the results of Matthew Cook. Finally we develop a relation between the models of John von Neumann, John Horton Conway and Cook, discussing important problems in cellular automata theory.



1 Historical background

A cellular automaton is a discrete-dynamical system which evolves through time, cellular automata theory may be divided in four stages of important contributions, the first with its precursor John von Neumann, the second with John Horton Conway, the third with Stephen Wolfram and we add a fourth stage with Matthew Cook.

In the middle of the 40's von Neumann was trying to develop a mechanism with two essential characteristics: support complex behaviors and the capacity of self-reproduction. Stanislaw M. Ullam, friend of von Neumann proposed him to take the idea into a mathematical model handling cells, in this way we have the rise of the cellular automata theory [35].

Von Neumann's model is a cellular automaton which evolves in two dimensions with twenty nine elements in the set of states and the transition function is defined by the von Neumann neighborhood, and with this model von Neumann demonstrates the feasibility of constructing a self-reproducing and an universal constructor system.

At the beginning of the 70's Conway presents a two-dimensional cellular automaton which is able to reproduce the same behaviors that the model of von Neumann, this automaton is better known as *The Game of Life*. But the difference of *Life* is that the set of states has only two elements and the transition function evolves with the Moore's neighborhood [8]. In 1982 it is demonstrated that *Life* can realize universal computation simulating a register machine by means of implementing logic gates [4].

In the middle of the 80's Wolfram makes a complete systematic study in one-dimensional cellular automata, where the set of states has an arbitrary number of elements greater than zero and the transition function depends on a central cell and its neighbors at both sides in a linear array.

Wolfram establishes a classification which has been projected to n-dimensional cellular automata [40]. This classification can be discussed in several aspects [21], [11] and [42], but we will just say that it is a phenotypical classification.

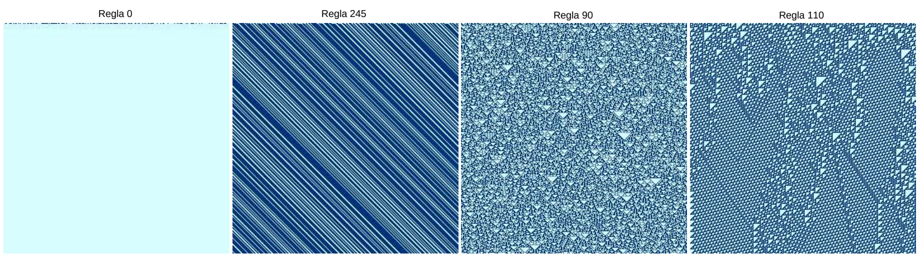


Figure 1: Wolfram classes

Class I represents stable behaviors, class II periodic behaviors, class III chaotic behaviors and class IV complex behaviors, as Figure 1 illustrates. From this classification, Wolfram conjectures that every cellular automaton class IV is able to realize universal computation, in particular he indicates in 1985 the cellular automaton Rule 110.

At the beginning of the 90's Cook resolves the conjecture of Wolfram demonstrating that Rule 110 is universal ([41] and [7]). This result has an outstanding importance within the history and the theory of cellular automata, because it is the simplest and the smallest cellular automaton fulfilling the characteristics established by von Neumann.

The result is not trivial and with theoretical interest, in this sense it is possible to establish a fourth stage where it is demonstrated that an elementary cellular automaton can make universal computation, and unlike the model of Conway, this automaton evolves in one dimension and with a linear transition function, but this point will be discussed in detail in the following section.

2 Preliminaries in Rule 110

The study of Rule 110 began with the first investigations of Wolfram about one-dimenional cellular automata.

Let us define some basic concepts used in the literature of cellular automata. Dynamics in one dimension is simple, we have a set of states $\Sigma \in \mathbb{Z}^+$, a finite linear array where each element takes a value from the set of states, this array is the initial configuration of the system. A neighborhood has a central cell and r neighbors on each side, where $r \in \mathbb{Q}$ and $k = |\Sigma|$; thus the neighborhood is of size $2r + 1$ and the transition function φ simultaneously evaluates each one of the k^{2r+1} neighborhoods throughout the array in each generation t, where t represents time. The previous concepts are illustrated in Figure 2.

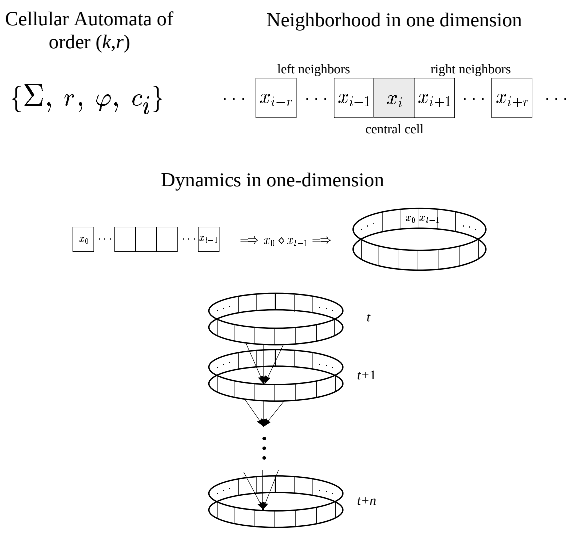


Figure 2: Dynamics in one dimension

Wolfram develops a complete systematic analysis in this type of cellular automata with order $(2,1)$, discovering that Rule 110 has complex behaviors and he establishes the conjecture that this rule can realize universal computation.

Rule 110 is a one-dimensional cellular automaton of order $(2,1)$ also called *elemental* by Wolfram, two states and a neighbor on each side. This automaton belongs to class IV because it has complex behaviors, that is, regions with stable, periodic and chaotic behaviors in the same evolution space. The evolution rule 110 (01110110_2) is defined for the transformation of each neighborhood: 000, 100 and 111 evolve into state 0 in the following generation and neighborhoods 001, 010, 011, 101 and 110 evolve into state 1.

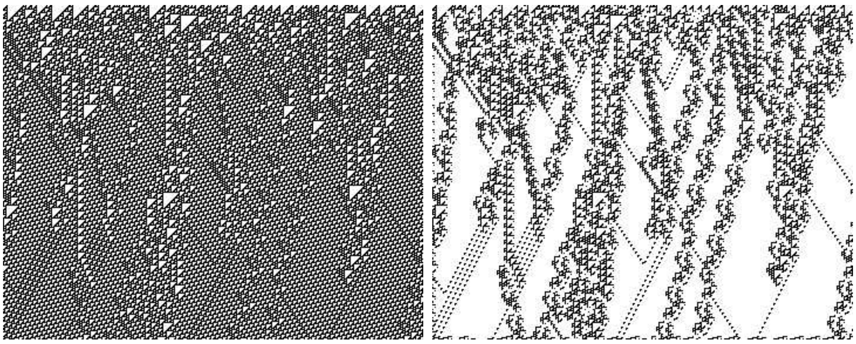


Figure 3: Random evolution in Rule 110

Figure 3 illustrates an example for the evolution of Rule 110 from a initial random configuration, we can see regions with stable behaviors (periodic background called *ether* by Cook), periodic regions represented by gliders, chaotic regions constructed by non-periodic structures during a long time which produces complex behaviors. The right figure is the same evolution but we have changed the ether colors which makes easier to identify gliders and chaotic constructions in the evolution space.

Cook specifies a classification of these periodic structures known in cellular automata literature as *gliders* [6], where glider means a periodic structure moving through time.

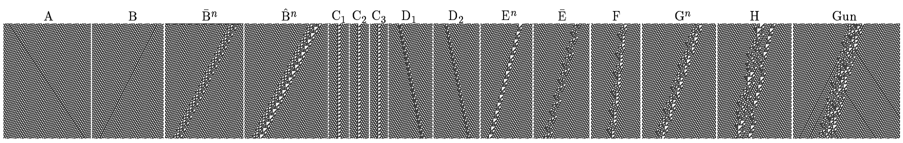


Figure 4: List of gliders according to the classification given by Cook

Figure 4 shows all the gliders until now known in Rule 110. Note that there exists gliders with shifts from right to left, from left to right and with no shift as the case of the C gliders. An interesting structure is the glider

Gun which generates A and B gliders periodically. Other relevant remark is the ample variety of options for constructing a glider Gun, among other complicated constructions in Rule 110 as we can see in [16].

The first article dedicated to the analysis of Rule 110 is by Wentian Li and Mats G. Nordahl in 1992 [20], where a statistical study is developed and some of the most common behaviors in the evolution space of Rule 110 are ilustrated.

In 1998 during a conference celebrated in Santa Fe Institute, Cook demonstrates how Rule 110 can realize universal computation. In January 1999 Cook presents a list of gliders found in the evolution space of Rule 110 [6]. At the present time this information is not available in Internet by legal problems, nevertheless these data can be consulted in [23], [41] and [15]. A good reference discussing this legal problem may be consulted in [9].

On the other hand, in 1999 Harold V. McIntosh develops an investigation based in Rule 110 is a problem of covering the evolution space with triangles [23].

Wolfram presented in March 2002 the book *A New Kind of Science*, where the cover is indeed an evolution of Rule 110. The book includes an ample variety of subjects and many illustrations, emphasizing the operation of an equivalent Turing system in Rule 110.

2.1 Results obtained by Li and Nordahl

Li and Nordahl apply the mean field theory for obtaining a general statistical description of the time behaviors in Rule 110 [20], this study is made in the evolution space in a global and nonlocal way, determining statistically the behavior of the space.

On the other hand, Doug Lind establishes a first glider classification that can be consulted in the appendix of [40], these gliders are those which naturally appear in the evolution space of Rule 110, identifying the periodic background and each one of the gliders to reproduce them in a particular way.

2.2 Results obtained by Wolfram

Wolfram discovers complex behaviors in Rule 110 and establishes the conjecture that this rule is able to realize universal computation, given the wide variety of gliders in its evolution space. Wolfram claims to have a new kind of science in its new book [41], this kind of science talks about when a simple system is able to reproduce highly complex behaviors.

The book discusses several subjects of general interest, nevertheless there are only about twelve pages dedicated to Rule 110, in them it is explained the universality of this rule and some complex behaviors like the list of gliders found by Cook. In october 2002 he offers a software *NKS Explorer* to reproduce any of the illustrations in the book, recently offers *A New Kind of Science: Notes*, a book available in Internet.

The book presents the operation of a cyclic tag systems but it does not give any antecedent or reference, and it is far from explain how the parts of the system and their global operation can be constructed. Something questionable is that the book claims copyright if someone tries to reproduce such a system. In this context it is better to discuss the universality of Rule 110 and other partial results as results of Cook.

2.3 Results obtained by Cook

We can say that the most important result after The Game of Life in cellular automata theory in the last twenty years, is the demonstration of the universality of Rule 110 made by Cook. The way in which this result is obtained devising and implementing a cyclic tag system is very ingenious and interesting.

This result can be seen in three fundamental parts, first in determining the system that must model a Turing machine [34], this model is represented as a variant of the tag system [28], [36], [37] and [29], the cyclic tag system [7]. The problem of deciding when a machine must stop in a tag system is the problem of the word correspondence proposed by Emil L. Post [31].

The second part is modeling the mechanism with the gliders of Rule 110, this is obtained handling well-defined blocks of gliders settling down a data area and another area of operations, implementing the basic operations of a tag system: read, erase and add data to the tape.

The third part is a consequence of the previous ones, describing the Turing machine in traditional terms identifying the reading head and the transformation rules.

Cook provides a list of gliders in the evolution space of Rule 110 [6] more complete than the one presented by Lind, leaving open the problem of finding more gliders. As in The Game of Life, it is interesting to know the number of structures that can exist in the evolution space, the list shows each one of the gliders and the extensions that some of them may have. This list has been reproduced through collisions and can be consulted in [15].

2.4 Results obtained by McIntosh

After the conference realized in Santa Fe Institute, McIntosh develops an investigation in Rule 110 raising the problem of covering the evolution space with triangles of different sizes [23].

This analysis offers several tools for the study, for example: the de Bruijn diagrams, the subset diagrams (or power set), the pair diagrams (or cartesian product), the cycle diagrams (or topologic trees), a matrix analysis and other tools like the mean field theory and block probabilities reflected as curves in the cartesian plane, contours or surfaces.

The problem to cover the evolution space with triangles is not mentioned by Cook or Wolfram, this approach allows to see the evolution space in a discrete way and it raises some interesting questions about finite shifts [10]. An interesting question is to know which is the largest triangle in the evolution space and if this one can be produced by some collisions as it is seen in [24]. His most recent publication discusses the universality of Rule 110 in [25].

2.5 Our results

Taking the list of gliders by Cook [6] and the analysis of McIntosh [23], we developed a systematic study in order to control the evolution space. The way as we do this is by means of the basic properties of the tile representing ether and these properties are reflected in all the structures of Rule 110.

Right now we do not use the probability tools as Li and Nordahl do; from the results of McIntosh we take the T_n tiles to get a discrete presentation of the evolution space through de Bruijn driagrams.

Unlike the analysis of Cook using HORIZONTAL and DIAGONAL measures by tile, we settle down a horizontal measurement called PHASE f_i_1 [13], this periodic phase is a regular expression that can be seen as a sequence of the extended de Bruijn diagram [22]. One problem with these diagrams is that on calculating more generations these grow exponentially and also the computing requirements. The phases calculated for larger gliders are obtained aligning the tile representing the periodic background in a phase f_i_1.

On the basis of this analysis, the computer system OSXLCAU21 was developed for the study of Rule 110 [44]. With this tool we offer an effective procedure to reproduce collisions between gliders (without extensions), and constructing initial configurations through of the concatenation of regular expressions.

3 Origin and fundamental concepts

In his work, von Neumann determines two essential characteristics supported by cellular automata: complex behaviors and self-reproduction. This takes von Neumann to raise two fundamental questions in cellular automata theory: How can we construct reliable components from nonreliable organisms?, and, What kind of logical organization is needed so that an automaton be able of self-reproduction?.

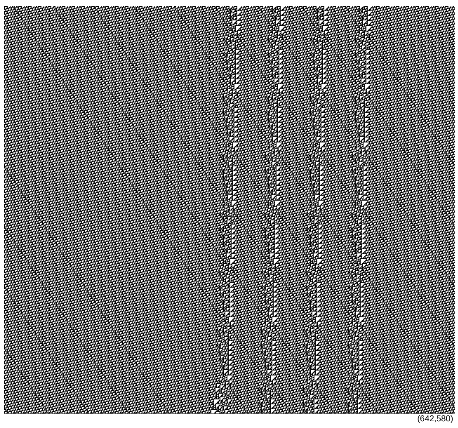


Figure 5: Synchronizing collisions among gliders

Both problems are complicated and represent extensive areas of study, this implies that we must try to synthesize these concepts as simple as possible. Finally we must relate these results with Rule 110. The cyclic tag system is a good example of constructing reliable components (each one of the devices constructed by Cook) from nonreliable organisms (gliders of Rule 110).

For example, Figure 5 illustrates the construction named meta-glider, through the synchronization several collisions among several gliders.

3.1 Complexity

Mathematical definitions about the complexity of systems come from computing and information theory. In the mathematical aspect, complexity is treated in the context of mathematical objects like sequences of characters. The *complexity* of a system is the amount of information necessary to describe it, this is a descriptive complexity. In dynamical systems the description includes the changes of the system in time influencing the environment. Then the amount of information necessary to describe this reaction is a system of complexity in its behavior (taken from Yaneer Bar-Yam [3]).

In cellular automata theory, complexity can be discussed in several contexts, but we will focus to the amount of information that can be produced in the evolution space. For instance in The Game of Life the number of interactions that we can have among gliders, still life or other structures grows exponentially. Rule 110 may support an infinite amount of information because the collision among them is not limited, for example there are gliders with extensions that belong to set of the positive numbers. On the other hand the formation of groups among several gliders originates a series of limitless collisions in Rule 110.

3.2 Self-reproduction and Universal Constructor

The idea of von Neumann was to imitate the behavior of the human brain for constructing a machine able to solve to complex problems [5]. He considered that a machine with such complexity must contain self-control and self-repair mechanisms. His idea was as well to establish differences between processes and data, considering that they are in equality. This drives to find a machine able to self-reproduction.

The first self-reproductive cellular automaton would be proposed by von Neumann evolving in a two-dimensional array, with twenty nine elements as a set of states and evolving with the von Neumann neighborhood. An important problem is the implementation of this model, due to this complexity the von Neumann rule has been partially implemented in a computer in [30] by U. Pesavento.

Von Neumann was successful in finding discrete structures of cells useful to generate new identical structures. Although this result is a very primitive form of life, it is very interesting because usually one hopes that a machine can only construct an object of smaller complexity than itself. With self-reproductive cellular automata we obtain a machine able to create new machines of identical complexity and capacities.

The von Neumann rule has in addition the property of making universal computation, this means that there is an initial configuration of the cellular automaton producing the solution of some algorithm, this property is of theoretical interest and not as much of practical application. Then the property of universal computation means that some computer logic (logic gates) can be simulated by a rule of a cellular automaton. All this demonstrates that the complex and unexpected behavior can emerge from a cellular automaton.

Von Neumann raises the idea of an universal constructor (taken from Barry McMullin [27] and [26]), in a sense which was not properly of computation. For this system he required of a decoder and a description in order to obtain the descendent phenomenon, in addition of requiring a copy of the description adding it as a part of the descendant.

This result is more difficult than demonstrate the existence of a simple self-reproducing machine, in particular it indicates the possibility that arbitrary complex machines are able of self-reproduction and this is what differences the von Neumann result from the one obtained by C. Langton in [19]. The operation of the machine of Langton can be divided in two activities: copy and decoder, because it does not incorporate something like a general constructive automaton.

Exactly self-reproduction is a characteristic of these complex systems, that is, systems which preserve but do not increase their level of complexity in their descendant systems. Von Neumann was interested in seeing exactly the minimum level of necessary organization for self-reproduction. As an index of this minimum complexity, he estimated the minimum size of patterns for the self-reproduction in a cellular space. Then it is possible to design an universal constructor in a rectangular adjustment of 57×143 cells, where several of the cells remain in a quiescent state.

The universal constructor concurs in a small part of the pattern for the self-reproduction. The reviewer unit and parts related to manipulation of memory are larger. Many of the patterns are given by the units coding and decoding the transmitted information or the reception of other parts of the pattern. Von Neumann estimated the total size of the patterns for self-reproduction in 200,000 cells, this order can vary depending on the design.

In 1982 Conway realized the penultimate reduction with the automaton The Game of Life, in two dimensions with a set of two states and evolving with the Moore neighborhood. With this automaton it was demonstrated a non-trivial reproduction similar to the von Neumann model. Nevertheless the rule of Conway was not designed to facilitate self-reproduction. The existence of self-reproducing patterns in the automaton of Conway is a strong evidence that the type of self-reproduction imagined by von Neumann is a natural phenomenon, possibly in many contexts [32].

In 1998 the last reduction was made with the demonstration of Cook in a one-dimensional cellular automaton, this automaton known as Rule 110 has two states and evolves in neighborhoods of size three [7].

The concept of universal constructor [26] is developed by Von Neumann, as a preliminary step to introduce the problem of obtaining spontaneous and open growth in the complex behavior of the automata. The concept was formulated as an analogy to the result of Turing and his universal machine.

The investigations made by von Neumann in the theory of complex automata are characterized by the problem of constructing non-trivial self-reproducing machines, where non-trivial is a requirement which means that the machine must have in an enviroment of a universal computer.

McMullin comments two very interesting things distinguishing an universal constructor from an universal machine. The Turing universal machine needs a description of the machine to work out, whereas the universal constructor is able to construct his own description and operate, that is, there are not Turing universal constructors.

Developing and obtaining a complete description of the universal constructor of von Neumann is really a very interesting and complicated task. On the other hand Conway raises an interesting problem in The Game of Life, it consisted of awarding the first person demonstrating a pattern with a constant and unlimited growth [32].

Many attempts were made to find such pattern, one of them was the arcon object formed by seven alive cells. Observing that it has a constant growth in a long term, but it was possible to check that its growth was not unlimited, finding stability in 5206 generations (nevertheless its evolution is very interesting). There were several techniques used to find such a pattern, however a group of students in MIT including R. William Gosper, Robert April, Michael Beeler, Richard Howell, Rich Schroeppel and Michael Specier solved the problem finding a generator of periodic structures known in cellular automata literature as a glider gun. In this sense a question is if von Neumann noticed this device in his model, something that surely must exist.

With this we conclude that an universal constructor must have conditions to reproduce anyone of its components in at least one way. An universal constructor must be able to maintain a constant and unlimited growth, and most contradictory of all these features is that an universal constructor is not able to construct everything. This last characteristic has been demonstrated in the model of Conway on finding a configuration that cannot be constructed by any way and it can just exist as an initial configuration. This configuration was found by Roger Banks using a mathematical technique to prove that there is a "Garden of Eden" configuration in a rectangular area of 9×33 cells [2].

3.3 Rule 110

Given the concepts explained in the previous section, it has been established a direct relation to study Rule 110. First we can say is that it is a cellular automaton of the von Neumann type, as well as the automaton of Conway. The von Neumann model is a two-dimensional cellular automaton with twenty nine states in the system and five cells for each transformation (29^5). The model of Conway is a two-dimensional automaton with two states and eight cells for each transformation (2^9). Rule 110 evolves in one dimension with two states and three cells in its transition rule (2^3).

The order of each model is really representative, nevertheless although Rule 110 has a smaller order than the model of von Neumann and Conway, the complexity that may be reproduced by this automaton is really complicated. In Rule 110 we can find each one of the elements to identify a cellular automaton of von Neumann type.

Rule 110 has an own universe originated by the variety of gliders that the system can produce and the number of interactions that they have, where this is an unlimited interaction. Since it had been mentioned, there are extensions and nonfinite groups of glides. For example, $\bar{B}^3$ glider at the present time is not produced through some way.

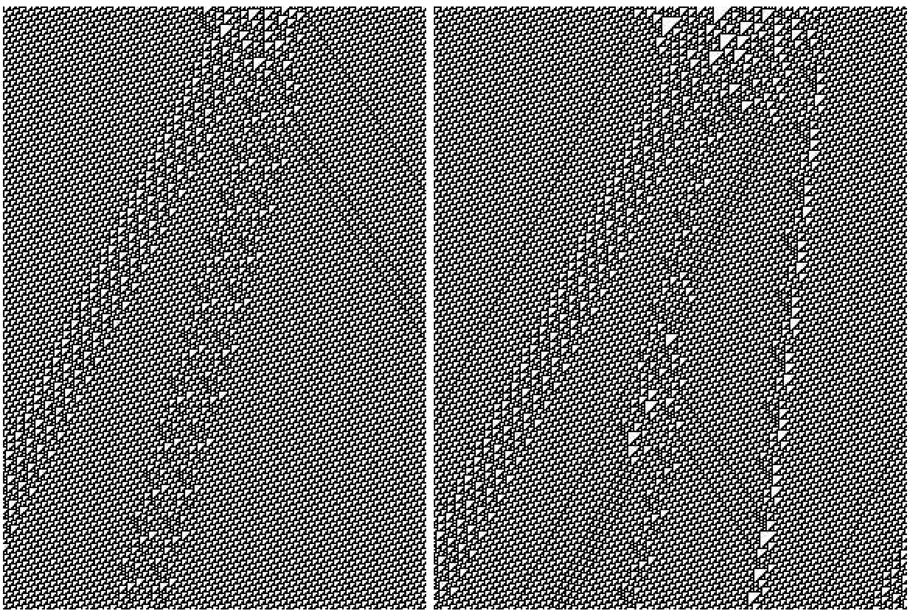


Figure 6: $\bar{B}^3$ glider

Figure 6 shows two examples rising a $\bar{B}^3$ glider,[1] the interest is if it is possibly reproduced through others gliders (with a collision) or is product of a configuration in the Garden of Eden. All gliders in Figure 4 has n extensions for every $n \geq 1$.

Rule 110 is able to yield an unlimited growth in an infinite space, Cook solves this problem on finding a glider gun as shown in Figure 4.

Rule 110 cannot construct it everything, for example the sequence (01010)* is a configuration that can just exist in the initial configuration, is a Garden of

[1] These examples were found in Rule 110 Winter WorkShop

Eden and therefore the evolution rule can never construct it in the evolution space.

Rule 110 is able to create mechanisms inducing computations and therefore it is a system which supports universal computation. Each one of the devices used by Cook in the simulation of the cyclic tag system is really complicated because the synchronization of several collisions between several different gliders in a huge space is one to one, a change in a single bit destroys all the system [17].

Rule 110 can support complex emergent behaviors in large scale both in a microscopic and macroscopic level. Rule 110 is able to construct reliable components from nonreliable organism and supporting self-reproduction.

Rule 110 has many similarities with The Game of Life, a interesting problem is to know some objects that can be useful to simulate computations or other phenomena and how they can be constructed.

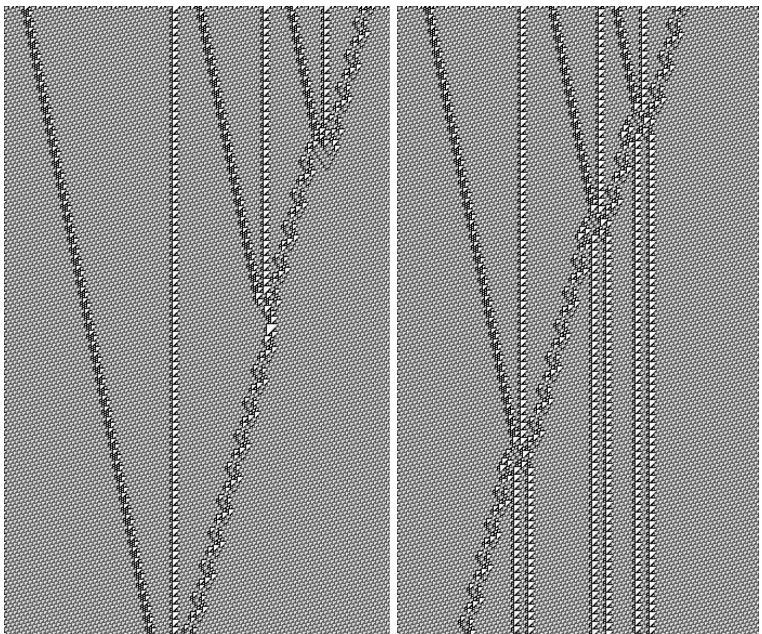


Figure 7: Eater and fuses objects

Figure 7 shows two new objects found in "Rule 110 Winter WorkShop." The first object can be seen as a eater glider, the $\bar{E}$ glider eliminates D_1 and C_2

gliders in each collision.

The second object can be seen as a shift between gliders, D_1 and C_2 gliders are changed by a pair of C gliders in each collision against an $\bar{E}$ glider. Some of the characteristics and similarities with The Game of Life are discussed by Cook in [6].

An interesting question is: is there some evolution which never becomes stable in Rule 110?.

On the other hand, Rule 110 can directly simulate some physical phenomena, for example the simulation of solitons between structures of different construction [12] and [18], in a completely deterministic atmosphere and without forcing the cellular automaton.

The features presented by Rule 110 to implement mechanisms realizing computations or some other process based on collisions among gliders can be as extensive as The Game of Life. In this direction we can see a large number of alternatives in collision-based computing, as we can see in the book of Andrew Adamatzky [1], for example the implementation of a Turing machine in the model of Conway made by Paul Rendell.

4 Conclusions

It has been limited the interest risen to analyze Rule 110, Santa Fe Institute offered a course for the analysis of Rule 110 in the summer of 2001.[2] On the other hand Fred Lunnon has implemented an algorithm to obtain each one of the collisions among gliders of Rule 110. Recently Mirko Rahn formally discussed the operation of the cyclic tag system with regard of a Turing machine [33], the approach is implementing a function accepting some Turing machine and a configuration in Rule 110 which simulates the calculation of such a machine.

Rule 110 in spite of being an elementary cellular automaton, has an infinite universe and this is quickly comprehensible by the number of behaviors that can exist in the evolution space, collisions among gliders form a limitless data base, for example the $\bar{B}^n$, $\hat{B}^n$, E^n and G^n gliders have a limitless number of collisions, without forgetting the grouping of gliders with or without extensions and the unlimited growth represented by the glider Gun, thus in this sense we have unlimited growth, insolubility and undecidable problems.

Another tool that offers many results is the cycle diagram described by Andrew Wuensche in [38] and [42], where the atractors cycles determine the periodic behavior for a sequence of certain length, the ancestor ramification illustrates the manifold for a given sequence and the leaves the configurations in the Garden of Eden. At the present time, Wuensche has found a very interesting cellular automaton with the same characteristics that the model of Conway and Cook in [43].

Other question in the case of the cyclic tag system is implementing some operations like the Fibonnaci sequence made by Paul Chapman (January 2003), the parenthesis balance, among other things.

[2] http://www.santafe.edu/sfi/education/reus/reus01/projects/binkowski.html

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References

[1] Andrew Adamatzky (Ed.), *Collision-Based Computing*, Springer, 2002. (ISBN 1-85233-540-8)

[2] S. Amoroso and G. Cooper, "The Garden-of-Eden theorem for finite configurations," *Proceedings of the American Mathematical Society*, 1970.

[3] Yaneer Bar-Yam, *Dynamics of Complex Systems*, Perseus Books, 1997. (ISBN 0-201-55748-7)

[4] Elwyn R. Berlekamp, John H. Conway and Richard K. Guy, *Winning Ways for your Mathematical Plays*, Academic Press, 1982 (ISBN 0-12-091152-3) vol. 2, chapter 25.

[5] Bastien Chopard and Michel Droz, *Cellular Automata Modeling of Physical Systems*, Collection Aléa Saclay, Cambridge University Press, 1998. (ISBN 0-521-46168-5)

[6] Matthew Cook, "Introduction to the activity of rule 110" (copyright 1994-1998 Matthew Cook), http://w3.datanet.hu/~cook/Workshop/CellAut/Elementary/Rule110/110pics.html, January 1999.

[7] Matthew Cook, "Universality in Elementary Cellular Automata," personal comunication.

[8] Martin Gardner, "Mathematical Games - The fantastic combinations of John H. Conway's new solitaire game Life," *Scientific American* **223**, pp 120-123, 1970.

[9] Jim Giles, "What kind of science is this?" *Nature*, **417** 216-218 (16 May 2002).

[10] Branko Grünbaum and G. C. Shephard, *Tilings and Patterns*, W. H. Freeman and Company, New York 1987. (ISBN 0-7167-1193-1)

[11] Howard A. Gutowitz, "Mean Field vs. Wolfram Classification of Cellular Automata," http://www.santafe.edu/~hag/mfw/mfw.html, November 1989.

[12] Genaro Juárez Martínez, “Solitones en el autómata celular unidimensional regla 110,” http://delta.cs.cinvestav.mx/˜mcintosh/oldweb/working.html, Abril 8, 2002.

[13] Genaro Juárez Martínez, “Phases f_{i}_1 in Rule 110,” preprint.

[14] Genaro Juárez Martínez and Harold V. McIntosh, “ATLAS: Collisions of gliders like phases of ether in rule 110,” http://delta.cs.cinvestav.mx/˜mcintosh/comun/s2001/s2001.html, August 2001.

[15] Genaro Juárez Martínez, Harold V. McIntosh and Juan Carlos Seck Tuoh Mora, “Production of gliders by collisions in Rule 110,” *Lecture Notes in Computer Science* **2801**, 175-182, 2003.

[16] Genaro Juárez Martínez, Harold V. McIntosh and Juan Carlos Seck Tuoh Mora, “Gliders in Rule 110,” preprint.

[17] Genaro Juárez Martínez, Juan Carlos Seck Tuoh Mora and Harold V. McIntosh, “Reproducing the cyclic tag systems developed by Matthew Cook with Rule 110 using the phases f_{i}_1,” preprint.

[18] Mariusz H. Jakubowski, Ken Steiglitz and Richard Squier, “Computing with Solitons: A Review and Prospectus,” *Multiple-Valued Logic*, Special Issue on Collision-Based Computing, vol. 6, Numbers 5-6, 2001. (ISSN 1023-6627)

[19] C. G. Langton, “Self-Reproduction in Cellular Automata,” *Physica D* **10**, pp 135-144, 1984.

[20] Wentian Li and Mats G. Nordahl, “Transient behavior of cellular automaton rule 110,” *Physics Letters A* **166**, 335-339, (1992).

[21] Harold V. McIntosh, “Wolfram’s Class IV and a Good Life,” *Physica D* **45**, pp. 105-121, 1990.

[22] Harold V. McIntosh, “Linear cellular automata via de Bruijn diagrams,” http://delta.cs.cinvestav.mx/˜mcintosh/oldweb/pautomata.html, 1991.

[23] Harold V. McIntosh, “Rule 110 as it relates to the presence of gliders,” http://delta.cs.cinvestav.mx/˜mcintosh/oldweb/pautomata.html, January 1999.

[24] Harold V. McIntosh, “A Concordance for Rule 110,” http://delta.cs.cinvestav.mx/˜mcintosh/ oldweb/pautomata.html, April 2000.

[25] Harold V. McIntosh, “Rule 110 Is Universal!,” http://delta.cs.cinvestav.mx/˜mcintosh/oldweb/ pautomata.html, June 30, 2002.

[26] Barry McMullin, "What is a Universal Constructor?," Presented at the Second European Conference on *Artificial Life*, Brussels, May 1993.

[27] Barry McMullin, "John von Neumann and the Evolutionary Growth of Complexity: Looking Backwards, Looking Forwards ...," Presented at *Artificial Life VII*, Portland, Oregon, 2000.

[28] Marvin Minsky, *Computation: Finite and Infinite Machines*, Prentice Hall, 1967.

[29] Manfred Kudlek and Yurii Rogozhin, "New Small Universal Post Machine," *Lecture Notes in Computer Science* **2138**, 217-227, 2001.

[30] U. Pesavento, "An implementation of von Neumann's self-reproducing machine," *Artificial Life* **2**, 337-354, 1995.

[31] Emil L. Post, "Formal reductions of the general combinatorial decision problem" *American Journal of Mathematics*, vol. 65, pp. 197-215, 1943.

[32] William Poundstone, *The Recursive Universe*, William Morrow and Company, Inc. New York, 1985. (ISBN 0-688-03975-8)

[33] Mirko Rahn, "Universalität in Regel 110," http://www.stud.uni-karlsruhe.de/~uyp0/uni110.foil.ps.gz, 18 März, 2003.

[34] Alan M. Turing, "On Computable numbers, with an application to the Entscheidungsproblem," *Proceedings of the London Mathematical Society*, Ser. 2, vol. 42, pp. 230-265, 1936. Corrections, Ibid, vol 43, pp. 544-546, 1937.

[35] John von Neumann, *Theory of Self-reproducing Automata*, University of Illinois Press, Urbana and London 1966.

[36] Hao Wang, "A Variant to Turing's Theory os Computing Machines," *Journal of Association for Computing Machinery* **4** 63-92, 1957.

[37] Hao Wang, "Tag Systems and Lag Systems," *Mathematische Annalen* **152** 65-74, 1963.

[38] Andrew Wuensche and Mike Lesser, *The Global Dynamics of Cellular Automata*, Santa Fe Institute Studies in the Sciences of Complexity, Addison-Wesley Publishing Company, July 1992. (ISBN 0-201-55740-1)

[39] Stephen Wolfram, "Universality and complexity in cellular automata," *Physica D* **10**, 1-35, 1984.

[40] Stephen Wolfram, *Theory and Aplications of Cellular Automata*, World Scientific Press, Singapore, 1986. (ISBN 9971-50-124-4 pbk)

[41] Stephen Wolfram, *A New Kind of Science*, Wolfram Media, Inc., Champaign, Illinois, 2002. (ISBN 1-57955-008-8)

[42] Andrew Wuensche, "Classifying Cellular Automata Automatically," *Complexity*, vol. 4, no. 3, 47-66, 1999.

[43] Andrew Wuensche, "Self-replication by glider collisions; the beehive rule," www.ddlab.com, 2004.

[44] "OSXLCAU21" is a public domain system developed for OPENSTEP, Mac OS X and Windows systems. The application and code source are available in http://delta.cs.cinvestav.mx/~mcintosh/comun/ s2001/s2001.html, August 2001; and http://www.rule110.org/downloads/.